

A bridge between unified cosmic history by $f(R)$ -gravity and Bionic system

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Recently, the cosmological deceleration-acceleration transition redshift in $f(R)$ gravity has been considered in order to address consistently the problem of cosmic evolution. It is possible to show that the deceleration parameter changes sign at a given redshift according to observational data. Furthermore, a $f(R)$ gravity cosmological model can be constructed in brane-antibrane system starting from the very early universe and accounting for the cosmological redshift at all phases of cosmic history, from inflation to late time acceleration. Here we propose a $f(R)$ model where transition redshifts correspond to inflation-deceleration and deceleration-late time acceleration transitions starting from a Bion system. At the point where the universe was born, due to the transition of k black fundamental strings to the Bion configuration, the redshift is approximately infinity and decreases with reducing temperature ($z \sim T^2$). The Bion is a configuration in flat space of a universe-brane and a parallel anti-universe-brane connected by a wormhole. This wormhole is a channel for flowing energy from extra dimensions into our universe, occurring at inflation and decreasing with redshift as $z \sim T^{4+1/7}$. Dynamics consists with the fact that the wormhole misses its energy and vanishes as soon as inflation ends and deceleration begins. Approaching two universe branes together, a tachyon is originated, it grows up and causes the formation of a wormhole. We show that, in the framework of $f(R)$ gravity, the cosmological redshift depends on the tachyonic potential and has a significant decrease at deceleration-late time acceleration transition point ($z \sim T^{2/3}$). As soon as today acceleration approaches, the redshift tends to zero and the cosmological model reduces to the standard Λ CDM cosmology.

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I. INTRODUCTION

Today cosmology is facing with two fundamental questions: what is inflation and what is late time acceleration? In other words, why and how do both the very early and the very late universe expand with acceleration? Extended theories of gravity, and in particular $f(R)$ gravity seems a reliable approach to reply these questions and unifying the early time inflation with late time acceleration era [1–6]. This model allows, in principle, to study the inflationary epoch, the transition to the non-phantom standard cosmology (radiation/matter dominated eras) and today observed dark energy epoch. For example, the evolution of quintessence/phantom dominated era in modified $f(R)$ gravity has been widely considered starting from very elementary models [7] up to more realistic approaches [8]. In Refs. [1, 6], it is demonstrated that this type of gravity unifies the early-time inflation with late-time acceleration and can be consistent with observational data. In another scenario, authors unified R^m inflation with late-time Λ CDM epoch in the modified $f(R)$ gravity. They showed that this model is consistent with local tests like Newton law, stability of Earth-like gravitational solution and very heavy mass for additional scalar degree of freedom [2]. Furthermore, a unified description of early-time inflation and late-time acceleration has been studied in a non-linear modified $f(R)$ Horava-Lifshitz gravity. It has been demonstrated that the cosmological dynamics details are generically different from those of other viable $f(R)$ models [3]. Finally, it has been shown that inflation and the late-time acceleration can be realized in a modified Maxwell- $f(R)$ gravity which is consistent with Solar System tests [4].

Furthermore, a class of $f(R)$ gravity models has been analyzed considering the Ricci scalar as a function of the redshift z . In this case the $f(R)$ gravity model reduces to Λ CDM at $z \simeq 0$. The observational viability of this class has been examined and the deceleration-acceleration transition redshift has been constrained by suitable sets of observational data [5]. However, an important question is on the origin of unified model and transition redshift in

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$f(R)$ gravity. In other words, the question is if there is a fundamental origin of reliable $f(R)$ gravity models that can be tested by observations.

A possible answer to this question can come from a brane-antibrane system. In fact, the evolution of this system can change the cosmological redshift and produce different epochs of cosmic history (see also [9] for a cosmographic analysis). When the k black fundamental strings transitioned to the BIon configuration, our observed universe is born and the redshift is approximately infinity ($z \sim \infty$). The BIon is a configuration in flat space of a universe-brane and a parallel anti-universe-brane connected by a wormhole [10, 11]. At this stage, large amount of energy is transferred from anti-brane to our own universe brane: consequently the redshift reduces and inflation occurs. At second stage, the wormhole dies and there is no channel for flowing energy into our universe. In this condition, inflation ends and the cosmological redshift results reduced with lower velocity. With decreasing separation distance between the two universe branes, a tachyon is originated. It grows and causes the formation of another wormhole. At this stage, the late time acceleration starts and the cosmological redshift accelerates to lower values.

The outline of the paper is as the following. In Sect. II, we will discuss a bridge between the inflationary stage in BIon system and $f(R)$ gravity. We will show that the redshift parameter in the $f(R)$ model depends on the evolution of the wormhole in BIon. In Sect. III, we study the behavior of redshift in the second stage when the wormhole dies and there is only a pair of universe brane and antibrane. In Sect. IV, we obtain the amount of redshift in third stage where a new tachyonic wormhole is formed between branes and accelerates the destruction of the two universes. In Sect. V, we discuss the results and draw the conclusion.

II. STAGE 1: THE COSMOLOGICAL REDSHIFT IN THE EARLY TIME INFLATION

Let us assume that there is only a k black fundamental string at the beginning of time. In our model, the universe is born at the point where the thermodynamics of k non-extremal black fundamental strings is matched to that of the BIon configuration. We will construct our $f(R)$ gravity model in BIon and discuss that cosmological redshift depends on the number of branes and the distance between them.

The supergravity solution for the k coincident non-extremal black F-strings lying along the z direction is:

$$\begin{aligned} ds^2 &= H^{-1}(-f dt^2 + dz^2) + f^{-1} dr^2 + r^2 d\Omega_7^2 \\ e^{2\phi} &= H^{-1}, \quad B_0 = H^{-1} - 1, \\ H &= 1 + \frac{r_0^6 \sinh^2 \alpha}{r^6}, \quad f = 1 - \frac{r_0^6}{r^6} \end{aligned} \quad (1)$$

From this metric, the mass density along the z direction can be found [12]:

$$\frac{dM_{F1}}{dz} = T_{F1} k + \frac{16(T_{F1} k \pi)^{3/2} T^3}{81 T_{D3}} + \frac{40 T_{F1}^2 k^2 \pi^3 T^6}{729 T_{D3}^2} \quad (2)$$

On the other hand, for finite temperature BIon, the metric is [11]:

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) + \sum_{i=1}^6 dx_i^2. \quad (3)$$

Choosing the world volume coordinates of the D3-brane as $\{\sigma^a, a = 0..3\}$ and defining $\tau = \sigma^0$, $\sigma = \sigma^1$, the coordinates of BIon are given by [10, 11]:

$$t(\sigma^a) = \tau, \quad r(\sigma^a) = \sigma, \quad x_1(\sigma^a) = l(\sigma), \quad \theta(\sigma^a) = \sigma^2, \quad \phi(\sigma^a) = \sigma^3 \quad (4)$$

and the remaining coordinates $x_{i=2,..6}$ are constant. The embedding function $l(\sigma)$ describes the bending of the brane. Let l be a transverse coordinate to the branes and σ be the radius on the world-volume. The induced metric on the brane is then:

$$\gamma_{ab} d\sigma^a d\sigma^b = -d\tau^2 + (1 + l'(\sigma)^2) d\sigma^2 + \sigma^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (5)$$

so that the spatial volume element is $dV_3 = \sqrt{1 + l'(\sigma)^2} \sigma^2 d\Omega_2$. We impose the two boundary conditions that $l(\sigma) \rightarrow 0$ for $\sigma \rightarrow \infty$ and $z'(\sigma) \rightarrow -\infty$ for $\sigma \rightarrow \sigma_0$, where σ_0 is the minimal two-sphere radius of the configuration. For this BIon, the mass density along the z direction can be obtained [11]:

$$\frac{dM_{BIon}}{dl} = T_{F1} k + \frac{3\pi T_{F1}^2 k^2 T^4}{32 T_{D3}^2 \sigma_0^2} + \frac{7\pi^2 T_{F1}^3 k^3 T^8}{512 T_{D3}^4 \sigma_0^4} \quad (6)$$

Comparing the mass densities for BIon to the mass density for the F-strings, we see that the thermal BIon configuration behaves like k F-strings at $\sigma = \sigma_0$. At the corresponding point, σ_0 has the following dependence on the temperature:

$$\sigma_0 = \left(\frac{\sqrt{kT_{F1}}}{T_{D3}}\right)^{1/2} \sqrt{T} \left[C_0 + C_1 \frac{\sqrt{kT_{F1}}}{T_{D3}} T^3 \right] \quad (7)$$

where $T_{F1} = 4k\pi^2 T_{D3} g_s l_s^2$, C_0 , C_1 , F_0 , F_1 and F_2 are numerical coefficients which can be determined by requiring that the T^3 and T^6 terms in Eqs. (2) and (6) agree. At this point, two universes are born, however the wormhole is not formed yet. The metric of these Friedman-Robertson-Walker (FRW) universes are:

$$ds_{Uni1}^2 = ds_{Uni2}^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dl^2), \quad (8)$$

The mass density of black F-string, BIon and two universes should be equal at the corresponding point:

$$\begin{aligned} \rho_{uni1} + \rho_{uni2} &= \frac{dM_{F1}}{dl} = \frac{dM_{BIon}}{dl} \rightarrow \\ 6H^2 &= T_{F1}k + \frac{16(T_{F1}k\pi)^{3/2}T^3}{81T_{D3}} + \frac{40T_{F1}^2 k^2 \pi^3 T^6}{729T_{D3}^2} \end{aligned} \quad (9)$$

where H is the Hubble parameter and has a following functional form with redshift [5]:

$$H(z) = H_0 \sqrt{\Omega_m(1+z)^3 + \log(\exp(1 - \Omega_m) + \delta z)} \quad (10)$$

where Ω_m and δ are constants. Solving Eqs. (9) and using (10), we obtain the cosmological redshift in terms of temperature:

$$z \simeq \frac{1}{H_0^{1/3}} \left(T_{F1}k + \frac{16(T_{F1}k\pi)^{3/2}T^3}{81T_{D3}} + \frac{40T_{F1}^2 k^2 \pi^3 T^6}{729T_{D3}^2} + \Omega_m - 1 \right)^{1/3} - 1 \quad (11)$$

At the beginning of time, it is $T = \infty \rightarrow z = \infty$. On the other hand, Eq. (11) shows that cosmological redshift depends on the temperature and decreases very fast.

After a short period of time, wormhole is formed between brane and antibrane due to the F-string charge and the universe enters a phase of inflation. Putting k units of F-string charge along the radial direction and using Eq.(5), we obtain [10, 11]:

$$l(\sigma) = \int_{\sigma}^{\infty} d\sigma \left(\frac{F(\sigma)^2}{F(\sigma_0)^2} - 1 \right)^{-\frac{1}{2}} \quad (12)$$

At finite temperature, BIon $F(\sigma)$ is given by

$$F(\sigma) = \sigma^2 \frac{4\cosh^2 \alpha - 3}{\cosh^4 \alpha} \quad (13)$$

where $\cosh \alpha$ is determined by the following function:

$$\cosh^2 \alpha = \frac{3 \cos \frac{\delta}{3} + \sqrt{3} \sin \frac{\delta}{3}}{2 \cos \delta} \quad (14)$$

with the definitions:

$$\cos \delta \equiv \bar{T}^4 \sqrt{1 + \frac{k^2}{\sigma^4}}, \quad \bar{T} \equiv \left(\frac{9\pi^2 N}{4\sqrt{3}T_{D3}} \right) T, \quad \kappa \equiv \frac{kT_{F1}}{4\pi T_{D3}} \quad (15)$$

In the above equation, T is the finite temperature of BIon, N is the number of D3-branes and T_{D3} and T_{F1} are the tensions of brane and fundamental strings respectively. Attaching a mirror solution to Eq. (12), we construct a wormhole configuration. The separation distance $\Delta = 2l(\sigma_0)$ is between the N D3-branes and N anti D3-branes for a given brane-antibrane wormhole configuration defined by the four parameters N , k , T and σ_0 . We have:

$$\Delta = 2l(\sigma_0) = 2 \int_{\sigma_0}^{\infty} d\sigma \left(\frac{F(\sigma)^2}{F(\sigma_0)^2} - 1 \right)^{-\frac{1}{2}} \quad (16)$$

In the limit of small temperatures, we obtain:

$$\Delta = \frac{2\sqrt{\pi}\Gamma(5/4)}{\Gamma(3/4)}\sigma_0 \left(1 + \frac{8}{27} \frac{k^2}{\sigma_0^4} \bar{T}^8\right). \quad (17)$$

Let us now discuss the non-phantom inflationary model in thermal BIon. For this, we need to compute the contribution of the Bionic system to the 4D-dimensional universe energy momentum tensor. The electromagnetic tensor for a Bionic system with N D3-branes and k F-string charges is [11],

$$\begin{aligned} T^{00} &= \frac{2T_{D3}^2}{\pi T^4} \frac{F(\sigma)}{\sqrt{F^2(\sigma) - F^2(\sigma_0)}} \sigma^2 \frac{4\cosh^2\alpha + 1}{\cosh^4\alpha} \\ T^{ii} &= -\gamma^{ii} \frac{8T_{D3}^2}{\pi T^4} \frac{F(\sigma)}{\sqrt{F^2(\sigma) - F^2(\sigma_0)}} \sigma^2 \frac{1}{\cosh^2\alpha}, \quad i = 1, 2, 3 \\ T^{44} &= \frac{2T_{D3}^2}{\pi T^4} \frac{F(\sigma)}{F(\sigma_0)} \sigma^2 \frac{4\cosh^2\alpha + 1}{\cosh^4\alpha} \end{aligned} \quad (18)$$

These equations show that with increasing temperature in Bionic system, the energy-momentum tensors decreases. This is because when spikes of branes and antibranes are well separated, wormhole is not formed and there is no channel for flowing energy from universe branes into extra dimensions and temperature is very high. However when two universe branes are close each other and connected by a wormhole, temperature reduced to lower values.

Now, we can discuss the $f(R)$ gravity model at finite temperature BIon system and obtain the explicit form of temperature and equation of state parameter ω . To this end, we use the approach in Ref.[5] to unify Bionic and $f(R)$ inflation and the three phases of the universe expansion. The key ingredient is to obtain the redshift transition from decelerated to accelerated behavior and viceversa. In general, the $f(R)$ gravity has the following action:

$$S = \int d^4x \sqrt{-g} \{f(R) + L_m\} \quad (19)$$

where L_m is the matter Lagrangian and g is the determinant of the metric tensor. Here the energy density ρ_{curv} and the pressure p_{curv} are [5]:

$$\begin{aligned} \rho_{curv} &= \frac{1}{f'(R)} \left\{ \frac{1}{2} [f(R) - Rf(R)] - 3H\dot{R}f''(R) \right\}, \\ p_{curv} &= -\frac{1}{2} [f(R) - Rf(R)] + 3H\dot{R}f''(R) + \ddot{R}f''(R) + \dot{R}[\dot{R}f'''(R) - Hf''(R)] \end{aligned} \quad (20)$$

where the prime denotes the derivative with respect to R and dot derivative with respect to time. Following [5], the Ricci scalar R can be written as a function of z :

$$R = 6H[(1+z)H_z - 2H], \quad (21)$$

Also, we have with respect to the redshift z :

$$\begin{aligned} f'(R) &= R_z^{-1} f_z, \\ f''(R) &= (f_{2z}R_z - f_z R_{2z}) R_z^{-3} \\ f'''(R) &= \frac{f_{3z}}{R_z^3} - \frac{f_z R_{3z} + 3f_{2z}R_{2z}}{R_z^4} + 3\frac{f_z R_{2z}^2}{R_z^5} \end{aligned} \quad (22)$$

The ansatz for $f(z)$ which results in the above expressions are [5]:

$$f(z) = f_0 + \frac{1}{1+z} + f_1(1+z) + f_2(1+z)^2, \quad (23)$$

To evaluate the derivatives of $f(R)$ in terms of the Hubble rate, we make use of the following identities:

$$\begin{aligned} \dot{R} &= -(1+z)HR_z, \\ \ddot{R} &= (1+z)H[HR_z + (1+z)(H_z R_z + HR_{2z})] \end{aligned} \quad (24)$$

For these considerations, we adopt a class of $f(R)$ gravity models which are consistent with the Solar System tests [13]. Then, using Eq. (20), the equation of state parameter is written as [5]:

$$\omega_{curv} = -1 + \frac{\ddot{R}f''(R) + \dot{R}[\dot{R}f'''(R) - Hf''(R)]}{\frac{1}{2}[f(R) - Rf'(R)] - 3H\dot{R}f''(R)} \sim \begin{cases} -z^7 & \text{for } z \rightarrow \infty \\ -1 + z^4 & \text{for moderate values of } z \\ -z^{-3} & \text{for } z \rightarrow 0 \end{cases} \quad (25)$$

On the other hand, using Eq. (18) and assuming that higher-dimensional stress-energy tensor has the following relation with energy density and pressure ($T_i^j = \text{diag}[\rho, -p, -p, -p, -\bar{p}, -p, -p, -p]$), we can obtain the equation of state on the universe brane of the finite temperature BIon system:

$$\omega_{BIon} = -\frac{4\cosh^2\alpha}{4\cosh^2\alpha + 1}(1 + l'^2) \quad (26)$$

We assume that the wormhole is created at $T = T_0$ and $\sigma = \sigma_0$ and it vanishes at $T = T_1$ and $\sigma_0 = 0$. In this period of time, we can write: $\sigma_0 = \frac{T - T_1}{T_0 - T_1}\sigma$. Using this result and Eq. (12), we obtain the Bionic equation of state in terms of temperature:

$$\omega_{BIon} = -\frac{4\cosh^2\alpha}{4\cosh^2\alpha + 1} \left(1 + \frac{T - T_1}{(T_0 - T)(T_0 - 2T_1 + T)} \right) \quad (27)$$

This equation indicates that the equation of state parameter is less than -1 during the inflation era and tends to values larger than negative ones for $T = T_1$. Assuming the Bionic equation of state parameter equal to equation of state of $f(R)$ given in Eq. (25) and using Eqs. (21-24), we can estimate the cosmological redshift in terms of temperature, that is :

$$z \sim \frac{\frac{\sqrt{3}}{6}T^8 + T^4 - 3\sqrt{3}}{T^4} \left(1 + \frac{T - T_1}{(T_0 - T)(T_0 - 2T_1 + T)} \right)^{1/7} \quad (28)$$

As it can be seen from this equation, the cosmological redshift was infinity at $T = T_0$ and reduced to lower values with decreasing temperature. Using this parameter and Eq.(10), we can calculate the Hubble parameter in terms of temperature:

$$H(z) \simeq H_0 \left\{ \Omega_m \left(\frac{\frac{\sqrt{3}}{6}T^8 + T^4 - 3\sqrt{3}}{T^4} \right)^3 \left(1 + \frac{T - T_1}{(T_0 - T)(T_0 - 2T_1 + T)} \right)^{3/7} + \log \left[\exp(1 - \Omega_m) + \delta \frac{\frac{\sqrt{3}}{6}T^8 + T^4 - 3\sqrt{3}}{T^4} \left(1 + \frac{T - T_1}{(T_0 - T)(T_0 - 2T_1 + T)} \right)^{1/7} \right] \right\}^{1/2} \quad (29)$$

According to this result, Hubble parameter is infinity at $T = T_0$; however, with decreasing temperature, the universe is accelerated to very low values in a short period of time. From this point of view, the behavior of this parameter is the same as we expected from the thermal history of the universe.

III. STAGE 2: THE COSMOLOGICAL REDSHIFT IN DECELERATION ERA

In this section, we propose a model that allows to consider the cosmological redshift in deceleration era of universe. In this stage, with decreasing temperature and distance between the two branes, the wormhole gives its energy to the branes and dies, a tachyon is born and the expansion of two FRW universes is controlled by the tachyonic potential between branes and evolves from deceleration to acceleration phase.

To construct a decelerated model, we consider a set of D3- $\overline{D3}$ -brane pairs in the background (5) which are placed at points $l_1 = l/2$ and $l_2 = -l/2$ respectively so that the separation between the brane and antibrane is l . For the

simple case of a single D3- $\overline{\text{D3}}$ -brane pair with open string tachyon, the action is [14]:

$$S = -\tau_3 \int d^9\sigma \sum_{i=1}^2 V(TA, l) e^{-\phi(\sqrt{-\det A_i})} \quad \text{where}$$

$$(A_i)_{ab} = (g_{MN} - \frac{TA^2 l^2}{Q} g_{Mz} g_{zN}) \partial_a x_i^M \partial_b x_i^N + F_{ab}^i + \frac{1}{2Q} ((D_a TA)(D_b TA)^* + (D_a TA)^*(D_b TA))$$

$$+ i l (g_{az} + \partial_a l_i g_{iu})(TA(D_b TA)^* - TA^*(D_b TA)) + i l (TA(D_a TA)^* - TA^*(D_a TA))(g_{bz} + \partial_b l_i g_{iu}). \quad (30)$$

and

$$Q = 1 + TA^2 l^2 g_{uu},$$

$$D_a TA = \partial_a TA - i(A_{2,a} - A_{1,a})TA, V(TA, l) = g_s V(TA) \sqrt{Q},$$

$$e^\phi = g_s (1 + \frac{R^4}{l^4})^{-\frac{1}{2}}, \quad (31)$$

Here ϕ , $A_{2,a}$ and F_{ab}^i are the dilaton field, the gauge fields and field strengths on the world-volume of the non-BPS brane, respectively; TA is the tachyon field, τ_3 is the brane tension and $V(TA)$ is the tachyon potential. The indices a, b denote the tangent directions of the D-branes, while the indices M, N run over the background ten-dimensional space-time directions. The Dp-brane and the anti-Dp-brane are labeled by $i = 1$ and 2 respectively. Then the separation between these D-branes is defined by $l_2 - l_1 = l$. Also, in writing the above equations we are using the convention $2\pi\alpha' = 1$.

Let us consider now, for the sake of simplicity, only the σ dependence of the tachyon field TA and set the gauge fields to zero. In this case, the action (30) in the region that $r > R$ and $TA' \sim \text{constant}$ simplifies to

$$S \simeq -\frac{\tau_3}{g_s} \int dt \int d\sigma \sigma^2 V(TA) (\sqrt{D_{1,TA}} + \sqrt{D_{2,TA}}) \quad (32)$$

where $D_{1,TA} = D_{2,TA} \equiv D_{TA}$, $V_3 = \frac{4\pi^2}{3}$ is the volume of a unit S^3 and

$$D_{TA} = 1 + \frac{l'(\sigma)^2}{4} + TA^2 l^2 \quad (33)$$

where the prime denotes a derivative w.r.t. its argument σ . A potential which has been used in most of papers is [15–17]:

$$V(TA) = \frac{\tau_3}{\cosh \sqrt{\pi} TA} \quad (34)$$

The energy momentum tensor is obtained from action by calculating its functional derivative w.r.t. the background ten-dimensional metric g_{MN} . The precise relation is $T^{MN} = \frac{2}{\sqrt{-\det g}} \frac{\delta S}{\delta g_{MN}}$. We get, [14],

$$T_i^{00} = V(TA) \sqrt{D_{TA}},$$

$$T_i^{zz} = -V(TA) \frac{1}{\sqrt{D_{TA}}} \left(TA^2 l^2 + \frac{l^2}{4} \right)$$

$$T_i^{\sigma\sigma} = -V(TA) \frac{Q}{\sqrt{D_{TA}}} \quad (35)$$

Now, using the above equations, we can obtain the equation of state parameter as:

$$\omega_{\text{brane-antibrane}} = -\frac{1 + TA^2 l^2}{1 + \frac{l^2}{4} + TA^2 l^2} \quad (36)$$

This equation indicates that the equation of state parameter is bigger than -1 in the range of $T_2 < T < T_1$ where T_1 and T_2 are temperatures at the beginning and end of deceleration. We assume that the velocity of branes is very low during this era and we can choose $TA' \sim \eta$, $l' \sim \frac{TA}{(1+TA)^2} \sim \frac{\eta}{(1+TA)^2}$ and $l \sim l_0 \left(1 - \frac{TA}{1+TA} + \varepsilon \right)$. Putting this equation of state parameter equal to the equation of state parameter in (25) (which corresponds to the $f(R)$ gravity

and can be applied for all the three phases) and using Eqs. (21- 24), we obtain the cosmological redshift in terms of the tachyon:

$$z \sim 1 - \left[\frac{(1 + TA)^2 + TA^2 l_0^2 ((1 + TA) - TA + \varepsilon(1 + TA))^2}{(1 + TA)^2 + TA^2 l_0^2 ((1 + TA) - TA + \varepsilon(1 + TA))^2 + \eta^2} \right]^{1/4} \quad (37)$$

Eq.(37) shows that cosmological redshift depends on tachyon and with increasing tachyonic potential reduces to lower values. Using Eq. (10), we can write the Hubble parameter in terms of tachyon, that is

$$H \sim H_0 \left\{ 2 - \left[\frac{(1 + TA)^2 + TA^2 l_0^2 ((1 + TA) - TA + \varepsilon(1 + TA))^2}{(1 + TA)^2 + TA^2 l_0^2 ((1 + TA) - TA + \varepsilon(1 + TA))^2 + \eta^2} \right]^{1/4} \right\}^3 \quad (38)$$

This equation indicates that with increasing tachyon, the Hubble parameter decreases and, at large values of tachyon, tends to $H = H_0$.

IV. STAGE 3: THE COSMOLOGICAL REDSHIFT IN LATE-TIME ACCELERATION

In the previous section, we considered the cosmological redshift in the condition that tachyon field grows slowly ($TA \sim t \sim \frac{1}{z}$) and we ignored $TA' = \frac{\partial TA}{\partial \sigma}$ and $\dot{TA} = \frac{\partial TA}{\partial t}$ in our calculations. In this section, we discuss that, with the decreasing of the separation distance between brane-antibrane, tachyon field grows very fast, and TA' and \dot{TA} are not negligible. This dynamics leads to the formation of another wormhole. In this stage, the universe evolves from the deceleration phase to the acceleration phase and consequently, late-time acceleration era of the universe accelerates and ends up in a big-rip singularity. In this case, the action (30) gives the following Lagrangian L :

$$L \simeq -\frac{\tau_3}{g_s} \int d\sigma \sigma^2 V(TA) (\sqrt{D_{1,TA}} + \sqrt{D_{2,TA}}) \quad (39)$$

where

$$D_{1,TA} = D_{2,TA} \equiv D_{TA} = 1 + \frac{l'(\sigma)^2}{4} + \dot{TA}^2 - TA'^2 \quad (40)$$

and we assume that $TA l \ll TA'$. Now, we study the Hamiltonian corresponding to the above Lagrangian. To derive this, we need the canonical momentum density $\Pi = \frac{\partial L}{\partial \dot{TA}}$ associated with the tachyon:

$$\Pi = \frac{V(TA) \dot{TA}}{\sqrt{1 + \frac{l'(\sigma)^2}{4} + \dot{TA}^2 - TA'^2}} \quad (41)$$

so that the Hamiltonian can be obtained as:

$$H_{DBI} = 4\pi \int d\sigma \sigma^2 \Pi \dot{TA} - L \quad (42)$$

By choosing $\dot{TA} = 2TA'$, we have:

$$H_{DBI} = 4\pi \int d\sigma \sigma^2 \left[\Pi (\dot{TA} - \frac{1}{2} TA') \right] + \frac{1}{2} TA \partial_\sigma (\Pi \sigma^2) - L \quad (43)$$

In this equation, we have, integrating by parts, the term proportional to \dot{TA} , indicating that the tachyon can be studied as a Lagrange multiplier imposing the constraint $\partial_\sigma (\Pi \sigma^2 V(TA)) = 0$ on the canonical momentum. Solving this equation yields:

$$\Pi = \frac{\beta}{4\pi \sigma^2} \quad (44)$$

where β is a constant. Using Eqs. (44) in (42), we get:

$$\begin{aligned} H_{DBI} &= \int d\sigma V(TA) \sqrt{1 + \frac{l'(\sigma)^2}{4} + \dot{T}A^2 - TA'^2 F_{DBI}}, \\ F_{DBI} &= \sigma^2 \sqrt{1 + \frac{\beta}{\sigma^2}} \end{aligned} \quad (45)$$

The resulting equation of motion for $l(\sigma)$, calculating by varying (45), is

$$\left(\frac{l' F_{DBI}}{4 \sqrt{1 + \frac{l'(\sigma)^2}{4}}} \right)' = 0 \quad (46)$$

Solving this equation, we obtain:

$$l(\sigma) = 4 \int_{\sigma}^{\infty} d\sigma' \left(\frac{F_{DBI}(\sigma')}{F_{DBI}(\sigma_0)} - 1 \right)^{-\frac{1}{2}} = 4 \int_{\sigma}^{\infty} d\sigma' \left(\frac{\sqrt{\sigma_0^4 + \beta^2}}{\sqrt{\sigma'^4 - \sigma_0^4}} \right) \quad (47)$$

This solution, for non-zero σ_0 , represents a wormhole with a finite size throat. However, this solution is not complete, because, the acceleration of branes is ignored. This acceleration is due to tachyon potential between branes ($a \sim \frac{\partial V(T)}{\partial \sigma}$). According to recent investigations [18], each of the accelerated branes and antibranes detects the Unruh temperature ($T = \frac{\hbar a}{2k_B \pi c}$). We will show that this system is equivalent to the black brane.

The equations of motion obtained from the action (45) is:

$$\left(\frac{1}{\sqrt{D_{TA}}} TA'(\sigma) \right)' = \frac{1}{\sqrt{D_{TA}}} \left[\frac{V'(TA)}{V(TA)} (D_{TA} - TA'(\sigma)^2) \right] \quad (48)$$

We can re-obtain this equation in accelerated frame from the equation of motion in the flat background of (5):

$$-\frac{\partial^2 TA}{\partial \tau^2} + \frac{\partial^2 TA}{\partial \sigma^2} = 0 \quad (49)$$

By using the following re-parameterizations

$$\begin{aligned} \rho &= \frac{\sigma^2}{w}, \\ w &= \frac{V(TA) \sqrt{D_{TA}} F_{DBI}}{2M_{D3-brane}}, \\ \bar{\tau} &= \gamma \int_0^t d\tau' \frac{w}{\dot{w}} - \gamma \frac{\sigma^2}{2} \end{aligned} \quad (50)$$

and doing the following calculations:

$$\left[\left\{ \left(\frac{\partial \bar{\tau}}{\partial \tau} \right)^2 - \left(\frac{\partial \bar{\tau}}{\partial \sigma} \right)^2 \right\} \frac{\partial^2}{\partial \tau^2} + \left\{ \left(\frac{\partial \rho}{\partial \sigma} \right)^2 - \left(\frac{\partial \rho}{\partial \tau} \right)^2 \right\} \frac{\partial^2}{\partial \rho^2} \right] TA = 0 \quad (51)$$

we have:

$$(-g)^{-1/2} \frac{\partial}{\partial x_\mu} [(-g)^{1/2} g^{\mu\nu}] \frac{\partial}{\partial x_\nu} TA = 0 \quad (52)$$

where $x_0 = \bar{\tau}$, $x_1 = \rho$ and the metric elements are obtained as:

$$\begin{aligned} g^{\bar{\tau}\bar{\tau}} &\sim -\frac{1}{\beta^2} \left(\frac{w'}{w} \right)^2 \frac{(1 - (\frac{w}{w'})^2 \frac{1}{\sigma^4})}{(1 + (\frac{w}{w'})^2 \frac{(1+\gamma^{-2})}{\sigma^4})^{1/2}} \\ g^{\rho\rho} &\sim -(g^{\bar{\tau}\bar{\tau}})^{-1} \end{aligned} \quad (53)$$

where we have used of previous assumption ($\frac{\partial TA}{\partial t} = \frac{\partial TA}{\partial \tau} = 2 \frac{\partial TA}{\partial \sigma}$).

Now, we can compare these elements with the line elements of a black D3-brane [19]:

$$ds^2 = D^{-1/2} \bar{H}^{-1/2} (-f dt^2 + dx_1^2) + D^{1/2} \bar{H}^{-1/2} (dx_2^2 + dx_3^2) + D^{-1/2} \bar{H}^{1/2} (f^{-1} dr^2 + r^2 d\Omega_5^2) \quad (54)$$

where

$$\begin{aligned} f &= 1 - \frac{r_0^4}{r^4}, \\ \bar{H} &= 1 + \frac{r_0^4}{r^4} \sinh^2 \alpha \\ D^{-1} &= \cos^2 \varepsilon + H^{-1} \sin^2 \varepsilon \\ \cos \varepsilon &= \frac{1}{\sqrt{1 + \frac{\beta^2}{\sigma^4}}} \end{aligned} \quad (55)$$

Putting the line elements of (55) equal to line elements of (53), we find:

$$\begin{aligned} f &= 1 - \frac{r_0^4}{r^4} \sim 1 - \left(\frac{w}{w'}\right)^2 \frac{1}{\sigma^4}, \\ \bar{H} &= 1 + \frac{r_0^4}{r^4} \sinh^2 \alpha \sim 1 + \left(\frac{w}{w'}\right)^2 \frac{(1 + \gamma^{-2})}{\sigma^4} \\ D^{-1} &= \cos^2 \varepsilon + \bar{H}^{-1} \sin^2 \varepsilon \simeq 1 \\ \Rightarrow r &\sim \sigma, r_0 \sim \left(\frac{w}{w'}\right)^{1/2}, (1 + \gamma^{-2}) \sim \sinh^2 \alpha \end{aligned} \quad (56)$$

The temperature of BIon is $T = \frac{1}{\pi r_0 \cosh \alpha}$, see [10] for details. Consequently, the temperature of the brane-antibrane system can be calculated as:

$$\begin{aligned} T &= \frac{1}{\pi r_0 \cosh \alpha} = \frac{\gamma}{\pi} \left(\frac{w'}{w}\right)^{1/2} \sim \\ &\frac{\gamma}{\pi} \left(\tanh \sqrt{\pi} T A + \frac{l' l'' + T A' T A''}{1 + \frac{l'(\sigma)^2}{4} + T A'^2} + \frac{\frac{\beta}{\sigma^3}}{1 + \frac{\beta}{\sigma^2}} \right) \end{aligned} \quad (57)$$

Because γ depends on the temperature, we can write:

$$\begin{aligned} \gamma &= \frac{1}{\cosh \alpha} \sim \frac{2 \cos \delta}{3\sqrt{3} - \cos \delta - \frac{\sqrt{3}}{6} \cos^2 \delta} \sim \\ &\frac{2 \bar{T}^4 \sqrt{1 + \frac{\beta^2}{\sigma^4}}}{3\sqrt{3} - \bar{T}^4 \sqrt{1 + \frac{\beta^2}{\sigma^4}} - \frac{\sqrt{3}}{6} \bar{T}^8 (1 + \frac{\beta^2}{\sigma^4})} \end{aligned} \quad (58)$$

Using Eqs. (57) and (58), we can approximate the explicit form of the temperature as:

$$T \sim \left(\frac{4\sqrt{3} T_{D_3}}{9\pi^2 N} \right) \frac{\sqrt[3]{\pi}}{\sqrt[6]{1 + \frac{\beta^2}{\sigma^4}}} \left(\tanh \sqrt{\pi} T A + \frac{l' l'' + T A' T A''}{1 + \frac{l'(\sigma)^2}{4} + T A'^2} + \frac{\frac{\beta}{\sigma^3}}{1 + \frac{\beta}{\sigma^2}} \right)^{-1/3} \quad (59)$$

This equation shows that with approaching two branes together and increasing tachyon, the temperature of the system decreases. This result is consistent with the thermal history of the universe since the temperature decreases with time. Now, we want to estimate the dependency of tachyon on time. To this end, we calculate the energy momentum tensors

and equation of state parameter. Using the electromagnetic tensors for black D3-brane[10], we obtain:

$$\begin{aligned}
T^{00} &= \frac{\pi^2}{2} T_{D3}^2 r_0^4 (5 + 4 \sinh^2 \alpha) \sim \frac{\pi^2}{2} T_{D3}^2 \left(\frac{w}{w'} \right)^{1/2} (9 + \gamma^{-2}) \sim \\
&\frac{\pi^2}{2} T_{D3}^2 \left(\tanh \sqrt{\pi} T A + \frac{l' l'' + T A' T A''}{1 + \frac{\nu(\sigma)^2}{4} + T A'^2} + \frac{\frac{\beta}{\sigma^3}}{1 + \frac{\beta}{\sigma^2}} \right)^{-1} \left(9 + \frac{2 \bar{T}^4 \sqrt{1 + \frac{\beta^2}{\sigma^4}}}{3 \sqrt{3} - \bar{T}^4 \sqrt{1 + \frac{\beta^2}{\sigma^4}} - \frac{\sqrt{3}}{6} \bar{T}^8 (1 + \frac{\beta^2}{\sigma^4})} \right) \\
T^{ii} &= -\gamma^{ii} \frac{\pi^2}{2} T_{D3}^2 r_0^4 (1 + 4 \sinh^2 \alpha) \sim - \left(1 + \frac{l'^2}{4} \right) \frac{\pi^2}{2} T_{D3}^2 \left(\frac{w}{w'} \right)^{1/2} (5 + \gamma^{-2}) \sim \\
&- \left(1 + \frac{l'^2}{4} \right) \frac{\pi^2}{2} T_{D3}^2 \left(\tanh \sqrt{\pi} T A + \frac{l' l'' + T A' T A''}{1 + \frac{\nu(\sigma)^2}{4} + T A'^2} + \frac{\frac{\beta}{\sigma^3}}{1 + \frac{\beta}{\sigma^2}} \right)^{-1} \times \\
&\left(5 + \frac{2 \bar{T}^4 \sqrt{1 + \frac{\beta^2}{\sigma^4}}}{3 \sqrt{3} - \bar{T}^4 \sqrt{1 + \frac{\beta^2}{\sigma^4}} - \frac{\sqrt{3}}{6} \bar{T}^8 (1 + \frac{\beta^2}{\sigma^4})} \right)
\end{aligned} \tag{60}$$

We assume that the wormhole is created at $T = T_2$ and $\sigma_0 = 0$ and will cause to destruction of universe at $T = T_{rip}$ and $\sigma_0 = \sigma$. In this period of time, we can write: $\sigma_0 = \frac{T_2 - T}{T_2 - T_{rip}} \sigma$. Using this and the tensor ($T_i^j = \text{diag} [\rho, -p, -p, -p, -\bar{p}, -p, -p, -p]$), we can calculate the equation of state parameter:

$$\omega_{Bion} = - \frac{(T_2 - T_{rip})(1 + \beta^2 + (T - T_2)^2) \left(5 + \frac{2 \bar{T}^4 \sqrt{1 + \frac{\beta^2}{\sigma^4}}}{3 \sqrt{3} - \bar{T}^4 \sqrt{1 + \frac{\beta^2}{\sigma^4}} - \frac{\sqrt{3}}{6} \bar{T}^8 (1 + \frac{\beta^2}{\sigma^4})} \right)}{(T - T_{rip})(T - 2T_{rip} + T_2) \left(9 + \frac{2 \bar{T}^4 \sqrt{1 + \frac{\beta^2}{\sigma^4}}}{3 \sqrt{3} - \bar{T}^4 \sqrt{1 + \frac{\beta^2}{\sigma^4}} - \frac{\sqrt{3}}{6} \bar{T}^8 (1 + \frac{\beta^2}{\sigma^4})} \right)} \tag{61}$$

This equation shows that for $\beta^2 > \frac{5}{4}$, the equation of state parameter is negative at the beginning of this era and less than -1 in the range of $T_{rip} < T < T_{rip}$. Putting this equation of state parameter equal to the equation of state parameter in (25) (which corresponds to unified $f(R)$ model and can be applied for all the three phases), we get:

$$z \sim \left[\frac{(T - T_{rip})(T - 2T_{rip} + T_2) \left(9 + \frac{2 \bar{T}^4 \sqrt{1 + \frac{\beta^2}{\sigma^4}}}{3 \sqrt{3} - \bar{T}^4 \sqrt{1 + \frac{\beta^2}{\sigma^4}} - \frac{\sqrt{3}}{6} \bar{T}^8 (1 + \frac{\beta^2}{\sigma^4})} \right)}{(T_2 - T_{rip})(1 + \beta^2 + (T - T_2)^2) \left(5 + \frac{2 \bar{T}^4 \sqrt{1 + \frac{\beta^2}{\sigma^4}}}{3 \sqrt{3} - \bar{T}^4 \sqrt{1 + \frac{\beta^2}{\sigma^4}} - \frac{\sqrt{3}}{6} \bar{T}^8 (1 + \frac{\beta^2}{\sigma^4})} \right)} \right]^{1/3} \tag{62}$$

and

$$H \sim H_0 \left[1 + \frac{(T - T_{rip})(T - 2T_{rip} + T_2) \left(9 + \frac{2 \bar{T}^4 \sqrt{1 + \frac{\beta^2}{\sigma^4}}}{3 \sqrt{3} - \bar{T}^4 \sqrt{1 + \frac{\beta^2}{\sigma^4}} - \frac{\sqrt{3}}{6} \bar{T}^8 (1 + \frac{\beta^2}{\sigma^4})} \right)}{(T_2 - T_{rip})(1 + \beta^2 + (T - T_2)^2) \left(5 + \frac{2 \bar{T}^4 \sqrt{1 + \frac{\beta^2}{\sigma^4}}}{3 \sqrt{3} - \bar{T}^4 \sqrt{1 + \frac{\beta^2}{\sigma^4}} - \frac{\sqrt{3}}{6} \bar{T}^8 (1 + \frac{\beta^2}{\sigma^4})} \right)} \right] \tag{63}$$

These equations indicate that the cosmological redshift and the Hubble parameter decrease with temperature and tend to the values of Λ CDM model ($z \sim 0$ and $H = H_0$) at big rip singularity. As it can be seen from the cosmological redshift in the three stages of universe, this parameter is infinity at the beginning, reduces very fast in the inflation era, decreases with lower velocity in the deceleration phase, reduces with higher rate at the late-time acceleration and finally tends to zero at the ripping time. This result is, in principle, in agreement with recent observational data.

V. DISCUSSION AND CONCLUSIONS

In previous sections, we construct an $f(R)$ gravity model starting from a brane-antibrane system. Such an approach allows to obtain a unified cosmic history which comprises inflation, deceleration and acceleration phases. This model,

in principle, can be compared with cosmological data and can be used to determine the ripping time. To this end, let us calculate the deceleration parameter in inflation and late time acceleration epochs. For this goal, we use the result obtained in [5]:

$$q = -1 + \frac{(1+z)[3\Omega_m(1+z)^2 + \delta/(\alpha + \delta z)]}{2[\Omega_m(1+z)^3 + \ln(\alpha + \delta z)]} \quad (64)$$

where $\alpha = \exp(1 - \Omega_m)$. Using the relation and Eqs. (28) and (62), the deceleration parameter at inflation and late time acceleration epochs are respectively

$$q \sim -\Omega_m \frac{\frac{\sqrt{3}}{6}T^8 + T^4 - 3\sqrt{3}}{T^4} \left(\frac{T-T_1}{(T_0-T)(T_0-2T_1+T)} \right)^{3/7} \left[1 + \frac{1}{\alpha + \delta \left(\frac{T-T_1}{(T_0-T)(T_0-2T_1+T)} \right)^{1/7}} \right] \quad \text{inflation era, } z \rightarrow \infty$$

$$q \sim -(\delta + 1) \frac{(T_2 - T_{rip})(1 + \beta^2 + (T - T_2)^2)}{(T - T_{rip})(T - 2T_{rip} + T_2)} \left[\alpha - \frac{2}{(T - T_{rip})(T - 2T_{rip} + T_2)} \right] \quad \text{acceleration era, } z \rightarrow 0 \quad (65)$$

It is easy to see that if we choose $\alpha = 2$, $\delta = 0.7$, $\Omega_m = 0.7$, $T_0 = 10^{32}$, $T_1 = 10^9$, $T_2 = 10^4$ and $T_{rip} = 0$, we find that $q = -0.542$ which leads to $T_{universe} = 3.5$ (in proper temperature units). This result is compatible with SNeIa data [20]. Besides, the deceleration parameter is negative in the range $T_1 < T < T_0$ and becomes zero at $T = T_1$. This means that the universe inflates in this period of time. On the other hand, q is zero at $T = T_2$ and negative again in today acceleration epoch, tending to $-\infty$ at big rip singularity.

In summary, the present $f(R)$ gravity model, derived from a brane-antibrane system, unifies inflation, deceleration and acceleration phases of expansion history. Specifically, the cosmological redshift z and the Hubble parameter can be obtained in terms of temperature T . These parameters decrease with reducing temperature and tend to the values of Λ CDM ($z = 0$, $H = H_0$) at present time. It is possible to show that, at transition point, a BIon system is formed due to the evolutions of black fundamental strings at transition point, $z \sim T^2$. This BIon system is a configuration of the parallel universe-brane and anti-universe-brane connected by a wormhole in flat space. With decreasing temperature, the energy of this wormhole flows into the universe branes and leads to inflation. We observe that, at inflation era, the cosmological redshift reduces very fast ($z \sim T^{4+1/7}$). After the death of the wormhole, inflation ends, deceleration epoch starts and cosmological redshift decreases with lower velocity. By approaching the two universe branes together, a tachyon is originated; it grows and causes the creation of a new wormhole. Late time acceleration era of the universe begins and, according to the model predictions, will end up in big-rip singularity. In this epoch, the cosmological redshift reduces very fast ($z \sim T^{1/3}$) and accelerated to zero. The model can be compared, in principle, with observational data and cosmological parameters can be obtained in terms of temperature and time. In a forthcoming paper, we will discuss the model along the track of Ref.[5] adopting recent observational data sets.

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